Schilling

OBSERVATIONES
IN QUOSDAM
SCRIPTORUM REI RUSTICAE VETERUM LATINORUM LOCOS.

PARTICULA PRIMA.

QUA PROBLEMATUM QUORUMDAM GEOMETRICORUM
DATAE A COLUMELLA RESOLUTIONES DEMONSTRANTUR.

SCRIPSIT

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SACRUM ESSE VOLEBAT

AUCTOR.

OBSERVATIONES
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PARTICULA PRIMA,

qua Problematum quae agunt de inveniendis trianguli aequilateri, item ſegmenti circuli areis, datae a Columella (de re ruſtica lib. V, 2, 5 et V, 2, 9) reſolutiones demonſtrantur.

I.

Si tribus paribus lateribus triquetrum metiri debueris, Columella (V, 2, 5) hanc formulam ut ſequaris ſuadet:

> „Eſto ager triangulus pedum quoquoverſus tricentorum. Hunc numerum in ſe multiplicato. Fiunt pedum nonaginta millia. Huius ſummae partem tertiam ſumito, id eſt triginta millia. item ſumito decumam, id eſt novem millia. utramque ſummam componito. Fiunt pedes triginta novem millia. Dicemus hanc ſummam pedum quadratorum eſſe in eo triquetro."

Celeberrimus Hambergerus, ex quo Io. Matth. Gesnerus *) quaeſierat: an problematis ſolutio haec a Columella propoſita demonſtrari poſſit? huius quaeſtionis contendit nullam dari ſolutionem, quia aſſertum (quod nempe $\frac{1}{3}$ atque $\frac{1}{10}$ □i la-

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*) vid. Scriptores rei ruſt. vett. lat. cur. I. M. Gesnero. Lipſ. 1735. Tom. I. pag. 566. item Scriptt. r. r. vett. lat. ed. I. Gottl. Schneider. Lipſ. 1794. Tom. II. Part. II. p. 242.

teris Δli aequilateri ſint areae trianguli aequales) non niſi vero eſſet proximum.

Ego vero ſolutionem a Columella propoſitam demonſtrari poſſe contendo.

Ut demonſtratio dilucidior, promptior et cognitu facilior ſit, quantitates et datas et inveniendas literis alphabeti inſignibimus, ac repraeſentabimus e. g. aream triquetri per literam A, latus triquetri aequilateri per literam a, latus duarum dimenſionum, ſeu latus in ſe multiplicatum per aa, vel a^2.

Quibus literis expreſſa formula ad inveniendam triquetri aequilateri aream a Columella propoſita haec eſt:

$$A = \frac{a^2}{3} + \frac{a^2}{10}.$$

Quam autem formulam mutari licet ſic:

$$A = a^2 \times \tfrac{1}{3} + a^2 \times \tfrac{1}{10}, \text{ et porro}$$

$$\text{cum ſint } \tfrac{1}{3} = 0{,}33333 \ldots \text{ et } \tfrac{1}{10} = 0{,}1$$

$$A = a^2 \times 0{,}33333 \ldots + a^2 \times 0{,}1$$

$$= a^2 \times (0{,}33333 + 0{,}1)$$

$$= a^2 \times 0{,}433333 \ldots .$$

Quaeritur: an haec formula

$$A = a^2 \times 0{,}43333 \ldots .$$

det in rigore geometrico aream trianguli aequilateri?

Ut inveniatur area trianguli cuiuslibet, didicimns ex elementis geometriae,

1) multiplicetur baſis per altitudinem; erit productum area rectanguli eiusdem baſeos et altitudinis;

2) productum dividatur bifariam. Ita prodit area trianguli.

Aliter: Baſis dimidia multiplicetur per altitudinem, vel baſis per altitudinem dimidiam. Factum erit area trianguli.

Poſitis baſi $= b$, altitudine $= a$,

$$A = \frac{a \times b}{2} = \frac{a}{2} \times b = a \times \frac{b}{2}.$$

Priusquam huius problematis reſolutione, quae in elementis geometriae demonſtratur, ad trianguli aequilateri, cuius tantummodo latera data ſunt numeris, aream inveniendam uti poſſumus, altitudo quae non eſt data, ratiocinando ex baſi aut latere trianguli nobis ſupputanda eſt.

Quodſi ponamus (cf. Fig. I.) baſin trianguli aequilateri $AB = a$, altitudo CD erit $= \sqrt{a^2 - (\frac{1}{2}a)^2}$, quod ex geometriae elementis facile demonſtrari poteſt.

Hanc autem formulam mutari ſic licet:

$$CD = \sqrt{a^2 - \tfrac{1}{4}a^2} = \sqrt{\tfrac{3}{4}a^2} = \tfrac{a}{2}.\sqrt{3}$$
$$= \frac{a}{2} \times 1{,}7320508 = a \times \frac{1{,}7320508}{2}$$
$$= a \times 0{,}8660254$$
$$\text{igitur } A = a \times a \times \frac{0{,}8660254}{2}$$
$$= a^2 \times 0{,}4330127.$$

Quod factum ſi conferamus cum formula quam Columella exhibet, quis eſt, qui non perſuaſum ſibi habeat, Columellae formulam rigori geometrico quam proximam et in geometria practica eſſe maximi momenti?

Dat enim Columellae formula tantummodo

$a^2 \times 0{,}0003206$ plus quam altera formula.

Quae differentia quam ſit leviſſima, intelliges, ubi pedibus trecentis, quae habent ſingula latera triquetri Columellae, addideris nonam pedis partem. Invenies enim, pedes tricentos cum nona pedis parte ſecundum formulam accuratiorem eandem dare aream, quam dant tricenti pedes ſecundum formulam Columellae.

Quis autem agrimenſor eſt, qui gloriari audeat, ſe in metienda tricentorum pedum linea numquam nona pedis parte erraſſe?

II.

Problematis de invenienda ſegmenti circuli area hanc dat Columella (*V*, 2, 9) *reſolutionem:*

„Si autem minus quam ſemicirculus erit (ſc. ager), arcum ſic metiemur: Eſto arcus, cuius baſis habeat pedes XVI, latitudo autem pedes IIII. Latitudinem cum baſi pono. Fit utrumque pedes XX. *Hoc duco quater.* Fiunt LXXX. Horum pars dimidia eſt XL. Item ſexdecim pedum, qui ſunt baſis, pars dimidia VIII. Hi VIII in ſe multiplicati fiunt LXIIII. Quartam decimam partem duco; ea efficit pedes IIII. paulo amplius. hoc adiicies ad quadraginta. fit utraque ſumma pedes XLIIII. Hos in arcu quadratos eſſe dico."

De huius quoque problematis ſolutione Hambergerus *) negat, eam usquam dare veram magnitudinem, niſi ubi chorda ſe habet ad ſagittam ut 16 ad 4, hinc neque demonſtrari poſſe.

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*) vid. Scriptt. r. r. vett. lat. ed. Gesner. Tom. I. pag. 535 et 568; item Scriptt. r. r. vett. lat. ed. Schneider. Tom. II. Part. II. pag. 245.

Sed errat doctiſſimus vir, qui illa Columellae verba: „*Hoc duco quater*," non bene intellexiſſe mihi videtur. Quae verba interpretor ſic:

„Hanc ſummam (ſc. pedes XX) multiplico per altitudinem, quae eſt pedum IIII."

Nullam aliam huius loci interpretationem admitti poſſe, ex demonſtratione, quae ſubſequitur, lucide patebit.

Denotatis quantitate chordae ſeu baſeos per c, ſagittae ſeu altitudinis per s, area ſegmenti per A, ex Columellae praeſcriptis emanat ad inveniendam ſegmenti cujuslibet aream formula haecce:

$$A = \frac{(c+s)\,s}{2} + \frac{(\frac{1}{2}c)^2}{14}$$

Hanc ſi formulam accurate atque geometricis, ut ita dicam, oculis perſcrutemur et membratim ſecemus, mox perſuaſiſſimum nobis erit, Columellam aut alium quempiam huius formulae auctorem, ſegmentum circuli conſideraſſe quaſi trapezium, cuius baſes parallelae ſint chorda altera, altera ſagitta et cuius altitudo ſagitta ſegmenti (cf. Fig. II); tunc autem quum aream ſegmenti paululum iuſto minorem invenerit, ſupplementum quoddam adieciſſe. Secunda formulae pars, in qua invenimus numerum 14, ſignificat, auctorem primam formulae partem cum Archimedis reſolutione problematis, ex data diametro ſemicirculi invenire aream, comparaſſe. Faciamus idem quod auctor noſter fecit. Comparemus formulam $\frac{(c+s)\,s}{2} + \frac{(\frac{1}{2}c)^2}{14}$ cum illa, quam Archimedes docuit ad inveniendam ſemicirculi aream, quae eſt $\frac{1}{2}\cdot(\frac{1}{2}d)^2\cdot\frac{22}{7} = (\frac{1}{2}d)^2\cdot\frac{22}{14}$.

Poſitis chorda c aequali diametro d et ſagitta $s =$ ſemidiametro $\frac{1}{2}d$, formulam $\frac{(c+s)\,s}{2}$ mutemus in $\frac{(d\,.\,\frac{1}{2}d)}{2}\cdot\frac{1}{2}d = \frac{1}{4}d^2 + \frac{1}{8}d^2$.

Quod ſi factum $\frac{1}{4}d^2 + \frac{1}{8}d^2$ auferamus ab area ſemicirculi $(\frac{1}{2}d)^2\cdot\frac{22}{14}$, habebimus

$$= (\tfrac{1}{2}d)^2\cdot\frac{22}{14} - (\tfrac{1}{2}d)^2 - \tfrac{1}{2}(\tfrac{1}{2}d)^2$$

$$= (\tfrac{1}{2}d)^2\left[\frac{22}{14} - 1 - \frac{1}{2}\right] = (\tfrac{1}{2}d)^2\left[\frac{22}{14} - \frac{3}{2}\right]$$

$$= (\tfrac{1}{2}d)^2\left[\frac{22}{14} - \frac{21}{14}\right] = (\tfrac{1}{2}d)^2\cdot\frac{1}{14} = \frac{(\frac{1}{2}d^2)}{14}.$$

Hinc

Hinc patet, factum ex formula $\left(\frac{d+\frac{1}{2}d}{2}\right)\frac{1}{2}d$ non dare aream ſemicirculi ſed deſiderari adhuc ſupplementum $\frac{(\frac{1}{2}d)^2}{14}$.

Quum vero, ſi ad ſegmentum reſpiciamus, *d* repraeſentet chordam *c*, illi primae parti $\frac{(c+s)s}{2}$ adhuc ſupplementum $\frac{(\frac{1}{2}c)^2}{14}$, ut area ſegmenti rectius atque perfectius exprimatur, addendum eſſe adparet.

Hoc modo illam, quam habet Columella, ad ſegmentum metiendum formulam exortam fuiſſe cenſeo.

Quae cum ita ſint, quis eſt qui perſuaſiſſimum ſibi non habeat, formulam illam $\frac{(c+s)s}{2}+\frac{(\frac{1}{2}c)^2}{14}$, quam ex formula $\frac{1}{2}\cdot(\frac{1}{2}d)^2\cdot\frac{22}{7}$ pro invenienda ſemicirculi area ab Archimede data, exortam vidimus, vice verſa ad metiendum ſemicirculum applicatam veram ſemicirculi aream eſſe daturam?

Sed quaeritur: an huius formulae fieri poſſit applicatio ad inveniendas minorum ſegmentorum areas?

Supputemus ad rigorem geometricum eorum quae maxime ſunt in promtu ſegmentorum minorum areas, et comparemus harum ſupputationum facta cum factis, quae e Columellae formula producuntur.

1. Segmentum cuius area ſupputatu eſt facillima, eſt graduum ſexaginta (conf. Fig. III.)

Ex elementis geometriae meminiſſe juvat, inveniri cuiuscunque ſegmenti minoris aream, ſi ex inveſtigata ſectoris area auferatur area trianguli duobus radiis et ſegmenti chorda comprehenſi; reſiduum erit ſegmentum.

a) Area ſectoris arcus 60° *ABCD* eſt ſexta pars circuli areae, quae eſt $=\frac{(\frac{1}{2}d)^2\cdot 22}{6\cdot 7}$.

b) Area ſegmenti arcus 60° *ABD* eſt $=\frac{(\frac{1}{2}d)^2\cdot 22}{6\cdot 7}-\Delta\,ABC$.

Cum $\Delta\,ABC$ eſt aequilaterum, cuius baſis eſt $\frac{1}{2}d$, area eius eſt, ut ſupra demonſtratum, $=(\frac{1}{2}d)^2\times 0{,}4330127$; hinc ſequitur, aream ſegmenti eſſe

$$ABD=\frac{(\frac{1}{2}d)^2\cdot 22}{6\cdot 7}-(\tfrac{1}{2}d)^2\times 0{,}4330127$$
$$=(\tfrac{1}{2}d)^2\times[0{,}5238095-0{,}4330127]$$
$$=(\tfrac{1}{2}d)^2\times 0{,}0907968.$$

Quod ut factum comparare poſſimus cum eo, quod formula Columellae dabit, inveſtiganda nobis eſt ſagittae *DE* quantitas, i. e. ſegmenti altitudo.

DE autem eſt $= CD - CE = \frac{1}{2}d - CE$.

Cum $\Delta\ ABC$ eſt aequilaterum, cuius latera ſingula ſunt $= \frac{1}{2}d$, eius altitudo CE, ut iam ſupra demonſtratum, eſt $= \frac{1}{2}d \times 0{,}8660254$.

Ergo $DE = \frac{1}{2}d - \frac{1}{2}d \times 0{,}8660254$
$= \frac{1}{2}d\,[1 - 0{,}8660254]$
$= \frac{1}{2}d \times 0{,}1339745$.

Nunc fiat applicatio ad formulam:

„Area ſegmenti $ABD = \frac{(c+s)\,s}{2} + \frac{(\frac{1}{2}c)^2}{14}$."

In noſtro hoc caſu chorda c eſt $=$ ſemidiametro $\frac{1}{2}d$, ſagitta s autem $= \frac{1}{2}d \times 0{,}1339745$; ergo

$$ABD = \left(\frac{\frac{1}{2}d + (\frac{1}{2}d \times 0{,}1339745)}{2}\right) \times (\tfrac{1}{2}d \times 0{,}1339745) + \frac{(\frac{1}{2} \times \frac{1}{2}d)^2}{14}$$
$$= (\tfrac{1}{2}d)^2\,\frac{[0{,}1339745 + 0{,}1339745^2]}{2} + \frac{(\frac{1}{2}d)^2}{4\,.\,14}$$
$$= (\tfrac{1}{2}d)^2\,\frac{[0{,}1339745 + 0{,}0179472]}{2} + (\tfrac{1}{2}d)^2 \times \frac{0{,}0357143}{2}$$
$$= (\tfrac{1}{2}d)^2\,[0{,}1876360] = (\tfrac{1}{2}d)^2 \times 0{,}0938180.$$

Hoc factum $= (\frac{1}{2}d)^2 \times 0{,}0938180$ ſi comparemus cum illo $= (\frac{1}{2}d)^2 \times 0{,}0907968$, quod rigori geometrico magis reſpondet, invenimus

$(\frac{1}{2}d)^2 \times 0{,}0030212$ differentiam,

qua ſecundum Columellae formulam area ſegmenti inventa eſt maior.

2. Iam inveſtigemus ſegmenti, cuius arcus eſt 90°, aream. (cf. Fig. IV.)

a) Area ſectoris $ADBC$ eſt quarta pars areae circuli $= \frac{(\frac{1}{2}d)^2\,.\,22}{4\,.\,7}$

b) Area triquetri rectanguli ACB eſt $= \frac{(\frac{1}{2}d \times \frac{1}{2}d)}{2}$

c) Ergo area ſegmenti $ABD = \frac{(\frac{1}{2}d)^2\,.\,22}{4\,.\,7} - (\frac{1}{2}d)^2\,.\,\frac{1}{2}$
$= (\frac{1}{2}d)^2\,.\,[\frac{22}{28} - \frac{1}{2}]$
$= (\frac{1}{2}d)^2\,.\,\frac{8}{28} = (\frac{1}{2}d)^2 \times \frac{2}{7}$
$= (\frac{1}{2}d)^2 \times 0{,}2857143$

Prinsquam hoc factum cum facto, quod e Columellae formula eruendum erit, comparari poteſt, et chorda AB et ſagitta DE inveſtigentur oportet.

Chorda AB eſt hypotenuſa triquetri rectanguli ACB, cuius cathetus eſt utraque $= \frac{1}{2}d$; ergo ſecundum theorema pythagoricum hypotenuſa

$$AB = \sqrt{(\tfrac{1}{2}d)^2 + (\tfrac{1}{2}d)^2}$$
$$= \sqrt{(\tfrac{1}{2}d)^2[1+1]} = \sqrt{(\tfrac{1}{2}d)^2 \times 2}$$
$$= \tfrac{1}{2}d\sqrt{2} = \tfrac{1}{2}d \times 1{,}4142135$$

ſagitta $DE = CD - CE = \tfrac{1}{2}d - CE$;

CE autem eſt $= \sqrt{CB^2 - (\tfrac{1}{2}AB)^2} = \sqrt{(\tfrac{1}{2}d)^2 - \frac{(\tfrac{1}{2}d)^2 \times 2}{4}}$
$$= \sqrt{(\tfrac{1}{2}d)^2 \cdot \times [1 - \tfrac{1}{2}]} = \tfrac{1}{2}d\sqrt{\tfrac{1}{2}}$$
$$= \tfrac{1}{2}d \times 0{,}707106.$$

Ergo $DE = \tfrac{1}{2}d - (\tfrac{1}{2}d \times 0{,}707106)$
$$= \tfrac{1}{2}d\,[1 - 0{,}707106]$$
$$= \tfrac{1}{2}d \times 0{,}292893.$$

Cum iam baſis $AB = \tfrac{1}{2}d \times 1{,}4142135$
ſagitta $DE = \tfrac{1}{2}d \times 0{,}292893$,
poſitis c pro AB, et s pro DE, area ſegmenti ABD, ſecundum formulam Columellae $\frac{(c+s)s}{2} + \frac{(\tfrac{1}{2}c)^2}{14}$ erit

$$ABD = \frac{(\tfrac{1}{2}d \times 1{,}4142135 + \tfrac{1}{2}d \times 0{,}292893)}{2}\,\tfrac{1}{2}d \times 0{,}292893 + \frac{\left(\frac{\tfrac{1}{2}d \times 1{,}414\ldots}{2}\right)^2}{14}$$
$$= (\tfrac{1}{2}d)^2\,[0{,}292893 \times 0{,}853553] + (\tfrac{1}{2}d)^2 \times 0{,}0357143$$
$$= (\tfrac{1}{2}d)^2 \times [0{,}25 + 0{,}0357143]$$
$$= (\tfrac{1}{2}d)^2 \times 0{,}2857143$$

Quod factum cum ſuperiori accuratiſſime congruit.

3. Nunc ſupputanda eſt area ſegmenti arcus 120 (cf. Fig. V).

a) Area ſectoris arcus 120° eſt tertia pars areae circuli $= \frac{(\tfrac{1}{2}d)^2 \cdot 22}{3 \cdot 7}$.

b) Area trianguli aequicruri ACB eſt aequalis areae trianguli aequilateri ACD, quod baſin habet ſemidiametrum $= (\tfrac{1}{2}d)^2 \times 0{,}4330127$. Quod ſic demonſtratur:

Linea CE perpendiculariter in chordam AB demiſſa atque ad peripheriam ducta et chordam AB et arcum ADB et angulum ad centrum biſariam ſecat, ita ut fiant $AE = BE$, $AD = BD$, $\angle ACD = \angle BCD = \angle 60^\circ$; conſequenter $\Delta\Delta\ ACD$ et BCD ſunt aequilatera et $\Delta AEC = \Delta BEC = \Delta BED = \Delta AED$;
ergo I) $DE = EC$
II) $\Delta AEC + \Delta BEC = \Delta AEC + \Delta AED$.
$\Delta ABC = \Delta ACD$ aequilatero.

Quod erat demonſtrandum.

c) Area ſegmenti *ABD* eſt

$$=\frac{(\frac{1}{2}d)^2 \cdot 22}{8 \cdot 7} - (\tfrac{1}{2}d)^2 \times 0,4330127$$
$$=(\tfrac{1}{2}d)^2 [1,04761905 - 0,4330127]$$
$$=(\tfrac{1}{2}d)^2 \times 0,6146063$$

Huius facti ut comparatio fieri poſſit cum illo quod e Columellae formula eſt exſpectandum, recordari iuvabit

I) ſagittam ſegmenti DE eſſe $= CE$

(quod ſupra eſt demonſtratum)

$$= \tfrac{1}{2} CD = \tfrac{1}{2}(\tfrac{1}{2}d) = \tfrac{1}{4}d.$$

II) chordam autem AB eſſe $= 2\,AE = 2\,BE$

$= \tfrac{1}{2}d \times 1,7320508$, quod ſic demonſtratur:

$$AE = \sqrt{AC^2 - EC^2} = \sqrt{(\tfrac{1}{2}d)^2 - (\tfrac{1}{2} \cdot \tfrac{1}{2}d)^2}$$
$$= \sqrt{(\tfrac{1}{2}d)^2 [1 - \tfrac{1}{4}]} = \tfrac{1}{2}d\sqrt{\tfrac{3}{4}} = \tfrac{1}{2}d \cdot \frac{\sqrt{3}}{2}$$
$$= \tfrac{1}{2}d \times \frac{1,7320508}{2} = \tfrac{1}{2}d \times 0,8660254$$

ergo $2\,AE$, ſive $AB = \tfrac{1}{2}d \times 1,7320508$

Quod erat demonſtrandum.

Repraeſentatis chorda per c et ſagitta per s, area ſegmenti ABD invenitur hoc modo:

$$ABD = \frac{(c+s)s}{2} + \frac{(\frac{1}{2}c)^2}{14}$$
$$= \frac{(\frac{1}{2}d \times 1,7320508 + \frac{1}{4}d)\,\frac{1}{4}d}{2} + \left(\frac{\frac{\frac{1}{2}d \times 1,7320508}{2}}{14}\right)^2$$

$$ABD = (\tfrac{1}{4}d \times 1,7320508 + 0,5)\,\tfrac{1}{4}d + (\tfrac{1}{2}d)^2 \tfrac{3}{56}$$
$$= (\tfrac{1}{2}d)^2 \times \frac{2,2320508}{4} + (\tfrac{1}{2}d)^2 \times 0,0535714$$
$$= (\tfrac{1}{2}d)^2 [0,5580127 + 0,0535714]$$
$$= (\tfrac{1}{2}d)^2 \cdot 0,6115841$$

Hoc factum $(\tfrac{1}{2}d)^2 \times 0,6115841$ e Columellae formula comparatum cum $(\tfrac{1}{2}d)^2 \times 0,6146068$

eſt $(\tfrac{1}{2}d)^2 \times 0,0030227$ minus.

Quibus ex demonſtrationibus patet, adhibita Columellae formula inveniri aream ſegmenti arcus $60^o = (\frac{1}{2}d)^2 \times 0{,}00302$ maiorem

90° veram

$120^o = (\frac{1}{2}d)^2 \times 0{,}00302$ minorem

180° veram,

ubi facta ex hac formula comparaveris cum productis, quae oriuntur ex Archimedis pro circulo metiendo formula data, poſita diametri ad peripheriam ratione ut 7 ad 22. Quam rationem 7 : 22, etſi non ſit ſatis accurata, his in demonſtrationibus et ſupputationibus adhibere non dubitavi, quia Columella nullam aliam peripheriae ad diametrum rationem cenſens accuratiorem, hac in ſupputationibus ſuis uſus eſt.

Denique nobis eſt perquirendum, quam accuratum ſit factum exempli a Columella exhibitum. Ducimur hac disquiſitione ad problema:

„Datis chorda et ſagitta, ſeu baſi et altitudine ſegmenti invenire aream eius."

Ex trigonometrie planae elementis meminiſſe iuvat, quomodo datis radio circuli et quantitate graduum arcus area ſegmenti inveniatur. Repraeſentatis radio per r, quantitate graduum per n, et area ſegmenti per A, eſt

$A = \frac{1}{2} r^2 (\frac{n\pi}{180} - \sin n)$ aut, ſi reſpiciamus ad diametrum d, cum r^2 ſit $= \frac{1}{4} d^2$,

$A = \frac{1}{8} (\frac{n\pi}{180} - \sin n) \times d^2$ I)

Poſitis (cf. Fig. VI) $DF = d$, $DE = s$, $AB = c$, $BE = \frac{1}{2}c$; datis s et c, ſupputetur diametri quantitas.

Quod ſit adhibita proportione continua hacce:

$$DE : BE = BE : EF,$$

i. e. $s : \frac{1}{2}c = \frac{1}{2}c : d - s$,

conſequenter $d - s = \dfrac{(\frac{1}{2}c)^2}{s}$

$$\text{et } d = \frac{(\frac{1}{2}c)^2}{s} + s \quad \text{II)}$$

DE autem eſt $= CD - CE = r - r\cos\frac{1}{2}n$

$= r(1 - \cos\frac{1}{2}n)$

aut $s = \frac{1}{2}d(1 - \cos\frac{1}{2}n)$

Poſito quoto $\frac{s}{d} = h$ III) erit

$$h = \frac{1}{2}d\frac{(1 - \cos\frac{1}{2}n)}{d} = \frac{1}{2}(1 - \cos\frac{1}{2}n)$$

conſequenter $2h = 1 - \cos\frac{1}{2}n$

ergo $\cos\frac{1}{2}n = 1 - 2h.$

Ex his ſequitur $\frac{1}{2}n$ eſſe arcum, cuius coſinus eſt $= 1 - 2h$, quod brevitatis cauſa ſic denotandum:

$$\tfrac{1}{2}n = \text{Arc.cof}(1-2h),\ \text{confequenter}\ n = 2\,\text{Arc.cof}(1-2h) \qquad \text{IV)}$$

Ut demonſtratio ſit dilucidior, omnia quae ſupra dicta ſunt, colligantur in regula ſequenti; ſecundum quam tunc Columellae exemplum ſupputandum erit:

„Datis chorda c et ſagitta s, inveſtigetur per aequationem II) diametros d, ſupputetur deinde ex III) quotus h. Quo invento conſtituatur arcus n ex IV) et denique ope I) area ſegmenti inquiratur."

Iam ad Columellae exemplum fiat huius regulae applicatio. Segmenti, quod Columellae in exemplo habet, baſis eſt pedum $XVI = c$, et altitudo pedum quatuor $= s$. Sequitur ex II), diametrum d eſſe $= \frac{8^2}{4} + 4 = 16 + 4 = 20$, ex III) $h = \frac{4}{20} = \frac{1}{5} = 0{,}2$, ergo $2h = 0{,}4$, et $1 - 2h = 1 - 0{,}4 = 0{,}6$ *).

Haec eſt (ſecundum IV) quantitas coſinus cuius arcum tabulae trigonometricae deſignant $= 53^\circ\, 7'\, 48''$; ergo

$$\begin{aligned} n &= 2 \times 53^\circ\, 7'\, 48 \\ &= 106^\circ\, 15'\, 36'' \\ &= 106{,}26^\circ \end{aligned}$$

Denique formulae I) haec eſt applicatio:

$$\begin{array}{rr} \log n = & 2{,}0263698 \\ +\log.\pi = & 0{,}4971499 \\ \hline & 2{,}5235197 \\ -\log.180 & 2{,}2552725 \\ \hline & 0{,}2682472 \end{array}$$

; cui logarithmo reſpondet

$$\begin{array}{rr} \text{numerus} & 1{,}854587 \\ -\sin n = & 0{,}960000 \\ \hline & 0{,}894587 \end{array}$$

, ducatur pars octava

8) 0,894587 / 0,111823, multiplicetur per $d^2 = 400$

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*) Si in formula III) pro d ſubſtituamus ex form. II) eius valorem $\frac{(\frac{1}{2}c)^2}{s} + s$, habebimus

$$h = \frac{s}{\frac{\frac{1}{4}c^2}{s} + s} = \frac{s^2}{\frac{1}{4}c^2 + s^2},\ \text{hinc}\ 2h = \frac{2s^2}{\frac{1}{4}c^2 + s^2}\ \text{et}\ 1 - 2h = \frac{\frac{1}{4}c^2 + s^2 - 2s}{\frac{1}{4}c^2 + s^2}.$$

Adhibita hac formula, nec diametro nec quoto h intervenientibus, n ex c et s ipſis inveniri poteſt; habebimus enim loco IV) hanc aequationem $n = 2\,\text{Arc.cof}\,\frac{\frac{1}{4}c^2 - s^2}{\frac{1}{4}c^2 + s^2}$; et poſito $\frac{1}{2}c = 8$, $s = 4$, erit $n = 2\,\text{Arc cof.}\,\frac{64-16}{64+16}$

$= 2\,\text{Arc.cof}\,0{,}6$, ut ſupra.

$A =$ 44,7292. Ad Columellae vero formulam eſt
$A =$ 44,5714, conſequenter

Differentia 0,1578.

Differentias, quas dat Columellae ad ſegmenta metienda formula, quam minimi et fere nullius eſſe momenti in geometria practica, perſuaſiſſimum nobis erit conſiderantibus, differentias illis multo maiores exorturas eſſe, ubi agrimenſor in metienda ſegmenti baſi modo paululum erraverit.

Igitur, quod demonſtrandum erat, ſcilicet:

„Columellam problemata, quae de areis et triquetri aequilateri et circuli ſegmentorum minorum inveniendis agunt, bene et ſimpliciter, nec non ad agrimenſorum uſum ſatis accurate reſolviſſe"

his paucis demonſtratum eſſe ſpero.

Fig. 1.

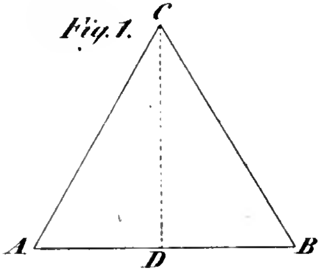


Fig. 2.

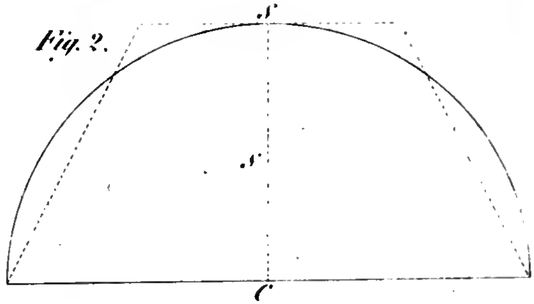


Fig. 3.

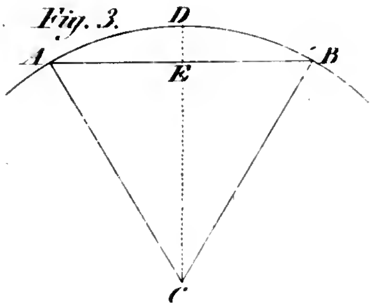


Fig. 4.

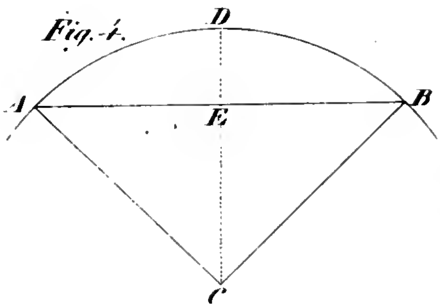


Fig. 5.

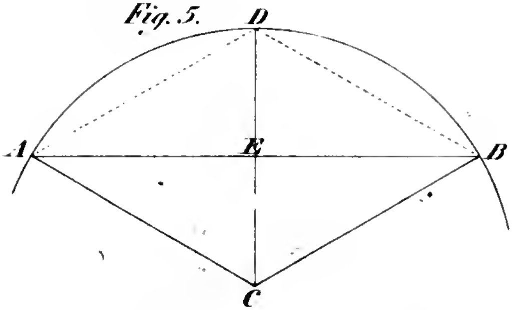


Fig. 6.

